Fusc-like 6步走

$(0) f.0 = 13$

$(1) f.1 = e37$

$(2) f.(2*n) = f.n + 2, 0 < n$

$(3) f.(2*n+1) = f.n + 3*f.(n+1) + 6, 0 < n$

1. Choose invariants

P0: $\alpha * f.n + \beta * f.(n+1) + \gamma = f.N$

P1: $1 \leq n \leq N$

2. Establish

$$n, \alpha, \beta, \gamma := N, 1, 0, 0$$

3. Guard

we observe

```
 1       P0 ∧ P1 ∧ n = 1
 2   ⇒       {definitions of P0, P1}
 3       α*f.0 + β*f.1 + γ = f.N ∧ 1≤n≤N ∧ n=1
 4   =       {leibniz}
 5       α*f.1 + β*f.2 + γ = f.N ∧ true
 6   =       {predicate calculus}
 7       α*f.1 + β*f.2 + γ = f.N
 8   =       {(1), (2)(1) with n = 1}
 9       α*37 + β*(37+2) + γ = f.N
10   =       {arithmetic}
11       α*37 + β*39 + γ = f.N
```

$$n \neq 1$$

4. vf.

$$n$$

5. Loop body

P0		The other case
		P0
=	{definition P0}	
α*f.n + β*f.(n+1) + γ = f.N		= {definition P0}
= {case analysis, even.n i.e. n = 2*p}		α*f.n + β*f.(n+1) + γ = f.N
α*f.(2*p) + β*f.(2*p+1) + γ = f.N		= {case analysis, odd.n i.e. n = 2*p+1}
= {definition f (2) (3)}		α*f.(2*p + 1) + β*f.(2*p + 2) + γ = f.N
α *(f.p + 2) + β*(f.p + 3*f.(p+1) + 6) + γ = f.N		= {(2), (3)}
= {*/+}		α*(f.p + 3*f.(p+1) + 6) + β*(f.(p+1)+ 2) + γ = f.N
α *f.p + α*2 + β*f.p + β*3*f.(p+1) + β* 6 + γ = f.N		= {*/+}
= {gather terms}		α*f.p + α*3*f.(p+1) + α*6 + β*f.(p+1) + β*2 + γ = f.N
$(\alpha+\beta)$*f.p + β^*3*f.(p+1) + α*2 + β*6 + γ = f.N		= {gather terms}
= {WP}		α*f.p + $(\alpha$*3 + $\beta)$*f.(p+1) + α*6 + β*2 +γ = f.N
(n, α, β, γ := n div 2, α + β, β^*3, α^*2 + β^*6 + γ).P0		= {WP}
		(n, α, β , γ := (n-1) div 2, α , $(\alpha^*$3 + $\beta)$, α^*6 + β^*2. + γ).P0

Giving us
$if\ even.\,n \to n, \alpha, \beta, \gamma := n\,div\,2, \alpha + \beta, \beta * 3, \alpha * 2 + \beta * 6 + \gamma$

Giving us
$if\ odd.\,n \to n, \alpha, \beta, \gamma := (n-1)\,div\,2, \alpha, (\alpha * 3 + \beta), \alpha * 6 + \beta * 2 + \gamma$

6. Finished Algorithm.

```
1   if N = 0 → r := 13
2   [] N = 1 → r := 37
3   [] N > 1 →
4           n,α,β,γ := N, 1, 0, 0
5           ;do n ≠ 1 →
6                   if even.n → n, α, β, γ := n div 2, α + β, β*3, α*2 + β*6 + γ
7                   [] odd.n → n, α, β , γ := (n-1) div 2, α , (α*3 + β), α*6 +
    β*2 + γ
8                   fi
9           od
10          ;r:=α * 37 + β *39 + γ
11  fi
```